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A model for the construction of a cost-minimal communication tree

By *Ludwig Nastansky* and *Hans Jürgen Drumm*

1. Introduction

For several years there have been increasing efforts to solve organizational problems by mathematical decision models. One can roughly distinguish three basic fields of research in solving organizational problems by means of quantitative methods. The first problem area includes personnel assignment problems which can be formulated and partly solved by procedures of mixed integer programming [1; 5; 4]. The second problem area contains problems of work flow planning and project management; usually they can be solved by network analysis techniques [3, pp. 58–80]. The third area has to do with the structure of communication systems and control of communication processes; these problems can be formulated by means of flow models and combinatorial algorithms; optimal solutions can be found under special conditions [9, pp. 140–161].

In this paper we consider the last problem area mentioned, yet we present a model of a very general type. This model is also appropriate for the solution of different problems, which do not belong to those of the communication system type [7]. Here our model will be used for the solution of what are called "structure-problems" in organizations. The model is based on graph theoretical methods; the basic structure of the optimizing problem belongs to a set of combinatorial procedures (cf. [2, chapt. II]).

First we develop the general organizational problem. Then the model will be formulated, followed by a small example and a summary of the solution procedure. In the appendix we shall demonstrate the efficiency of the model by help of three different exemplary decision situations. The first example deals with the diffusion of information on several decision-, staff- and executing units or stations. The second example deals with the construction of a management information system using a specific number of terminals. The third example demonstrates, how a new product can be optimally introduced through several different distribution channels.

2. The organizational problem

Usually the units or stations of an organization are linked by a set of communication paths. If a specific information has to be transmitted to one or several stations, the source of information can use a direct or an indirect – if not even several – communication path. The choice of a certain path depends on several items as for example the cost of communication, the term of communication or the capacity of the communication means. Moreover it can be necessary to include certain stations into a communication path, if official channels must be used, or if the information has to reach some stations in any case. Usually cost can be attributed to every communication path. It can therefore be sensible to choose the communication paths such that total cost of communication will be minimized. So the point is to select one or several cost-minimal communication paths out of a set of feasible communication paths. The following constraints must be satisfied:

(a) A certain number of stations must belong to the optimal path since they have to be informed about the communication process.

(b) Some communication paths cannot be used if a minimum speed of communication cannot be reached, or if the information cannot be transmitted within a feasible intervall of time.

(c) Some communication paths cannot be used, if they don't have a certain minimum-capacity.

If all communication paths, not satisfying constraints (b) and (c), are eliminated, our organizational problem can be formulated by means of the graph theory and solved by the algorithm subsequentyl described.

3. The decision model

3.1 The construction of the communication network

In the model we have a source of information S. The set S may consist of one or more different stations. Information has to be transmitted to recipients of information. All these receiving stations are defined as set T. It makes no difference whether the stations of set T only receive information from S or whether they also forward received information to subsequent stations. In the latter case, the stations act as recipients of information as well as relay stations for informations.

Beside the sets S and T we have the set B of pure relay stations. These stations receive informations only to transmit them. They can be used for the transmission of information, but they do not have to be.

Information has to be transmitted from the source of information to the recipients of information. We suppose any pair i, j [$i \neq j$; $i, j \in T \cup R$] of relay stations to be in an unequivocal hierarchical order. This means: if there is any path of communication between station i and station j there is only one feasible direction

for the flow of information over all possible paths between station i and station j. Thus, we exclude the possibility of a station to be located either before or after another station at the end of the decision process. We define all stations of the sets S, T, and R to be the nodes of a graph G. The set of all nodes of G is $N = S \cup T \cup R$. We assign an arc (i, j) to any possible case of information transmission from a station $i \in N$ to a station $j \in N$ [$i \neq j$]. The totality of these possible elementary ways of information transmission is defined as the arc set A of G. The cost c_{ij} of elementary communication from i to j are given for each arc (i, j) $\in A$ by the function C. The quantities determining the c_{ij} depend on the specific situation for which the model is to apply. If the problem is to construct a new communication network we have to consider cost which are fixed cost in the sense of short-term periodical planning. If we want to solve the problem of using an existing communication network in a cost-minimal way, c_{ij} will consist of variable cost components. As a consequence of this cost interpretation, the c_{ij} will be non-negative and naturally greater than zero.

The result of these definitions is the directed costdigraph G = (N, A, C). G has no (directed) cycles. This follows from the assumption of unequivocal hierarchy. Thus, a node indexing is possible such that for each arc $(i, j) \in A$ we have $i < j$. This is often advantageous for the purpose of simplifying bookkeeping in algorithms. From now on G will be called communication network.

3.2 The formulation of the optimization problem

The graph G represents all possible, technically feasible ways of information transmission for the underlying problem, and it contains the cost of elementary information transmission. The problem is now to extract a cost-minimal communication tree out of the communication network; i.e. we have exactly one path of communication from the source of information to each recipient of information.

Frequently this kind of a combinatorial decision model can be formulated as a linear (0,1)-programming problem. For this purpose, we assign a variable x_{ij} to each arc $(i, j) \in A$. $x_{ij} = 1$ says that we use the elementary way of communication transmission from station i to station j in the communication network, $x_{ij} = 0$ means that we do not use this elementary way.

P_j denotes the set of all direct predecessor nodes of station $j \in N$. S_j is the set of all direct successor nodes of station $j \in N$. The cardinality $|S_j|$ is the number of all direct successors of j.

The objective function of the linear (0,1)-programming problem is:

$$(1-1) \quad \sum_{(i,j) \in A} c_{ij} \, x_{ij} \quad \text{Min!}$$

The constraints are:

(1–2) $\sum_{i \in P_j} x_{ij} = 1 \qquad [j \in T]$

(1–3) $|S_j| \sum_{i \in P_j} x_{ij} \geqslant \sum_{k \in S_j} x_{jk} \qquad [j \in R]$

(1–4) $x_{ij} \in \{0,1\} \qquad [(i,j) \in A]$

Constraint (1–2) guarantees that each station j of the set T receives informations by means of a communication path from the source S. Constraint (1–3) asserts that there is no path of communication transmission leading out of a node j when there is no path leading to j. Moreover, this constraint allows, that all ways of information transmission leading out of node j (there are $|S_j|$ such ways) can be used if there is only one path leading to j. Constraint (1–3) is not necessary for stations $j \in T$ because constraint (1–2) guarantees for these stations in any case that a communication path is leading to them.

The objective function (1–1) together with the constraints (1–2) through (1–4) assert that the optimal solution of the linear (0,1)-programming problem defines a communication tree from the source of information to all recipients of information. The leaves of the tree are all recipients of information which only receive and do not forward information. It is possible to have a kind of degeneracy not resulting in a (0,1)-solution defining a tree because we allowed arcs with zero cost. In this case, we only have to remove some of the arcs with zero cost from the directed graph defined by the (0,1)-solution until the graph turns into a tree. This is a simple bookkeeping procedure but no optimization problem. As far as formulation and interpretation are concerned our problem of constructing a cost-minimal communication tree is quite similar to the ordinary problems of finding a cost-minimal flow in networks. The basic difference between the two problems is that in our problem we distinguish only the cases "information flow yes" (associated variable equal to one) and "information flow no" (associated variable equal to zero) for each elementary way of information transmission.

Once we have the case "information flow yes" for one elementary way of information transmission to a relay station we can forward the informations from this relay station so subsequent stations without any influence on the quantity of the information flow to this relay station. As a consequence, the cost of information transmission to a station do not depend on whether or not information is forwarded from this station. The flow conservation property of the ordinary network flow model, however, gives a different approach. For any unit of flow which we want to transport out of a node we have to assert one unit of flow being transported to this node. Here, as a consequence, the flow cost to this node are increased by

the unit cost of all arcs used to transmit the additional unit of flow from a source to this node.

3.3 A small example

We consider the example of a communication network given in Fig. 1.

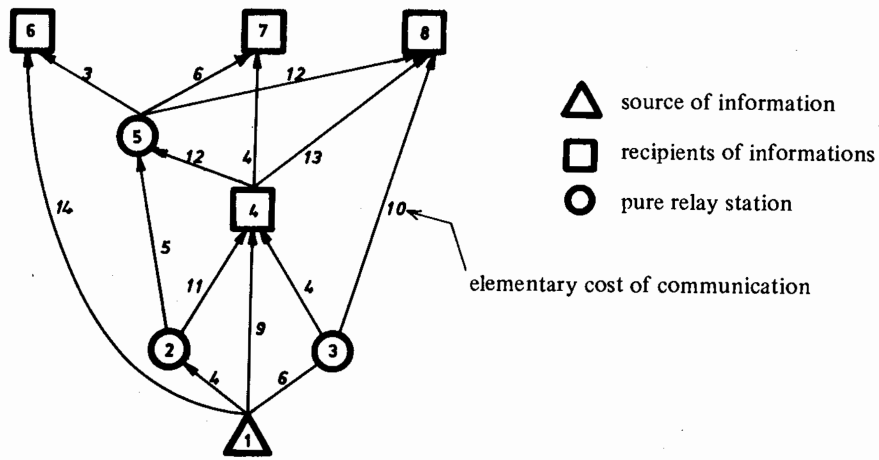


Fig. 1. An example of a communication network

The associated sets N and A of G = (N,A,C) are:

$$N = S \cup T \cup R$$
$$S = \{1\}$$
$$T = \{4, 6, 7, 8\}$$
$$R = \{2, 3, 5\}$$
$$A = \{(1,2),(1,3),(1,4),(1,6),(2,4),(2,5),(3,4),(3,8),(4,5),(4,7),(4,8),(5,6),(5,7),(5,8)\}$$

Examples for the sets P_j and S_j are:

$P_5 = \{2, 4\}$

$S_4 = \{5, 7, 8\} \qquad |S_4| = 3$

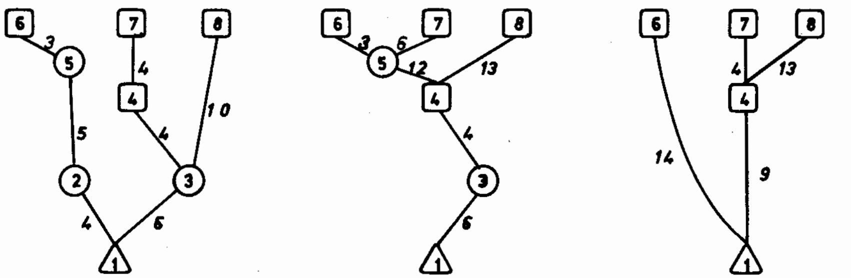
Explicitly the objective function of the associated linear (0, 1)-program reads:

$$
\begin{aligned}
(1-1)' \quad & 4x_{12} + 6x_{13} + 9x_{14} + 14x_{16} \\
& + 5x_{25} + 11x_{24} \\
& + 4x_{34} + 10x_{38} \\
& + 12x_{45} + 4x_{47} + 13x_{48} \\
& + 3x_{56} + 6x_{57} + 12x_{58} \quad \rightarrow \text{Min!}
\end{aligned}
$$

This objective function has to be minimized subject to:

$$
\begin{aligned}
(1-2)' \quad & \text{source of information } 4: && x_{14} + x_{24} + x_{34} = 1 \\
& \text{source of information } 6: && x_{16} + x_{56} = 1 \\
& \text{source of information } 7: && x_{47} + x_{57} = 1 \\
& \text{source of information } 8: && x_{38} + x_{48} + x_{58} = 1 \\
(1-3)' \quad & \text{pure relay station } 2: && 2x_{12} \geqslant x_{24} + x_{25} \\
& \text{pure relay station } 3: && 2x_{13} \geqslant x_{34} + x_{38} \\
& \text{pure relay station } 5: && 3x_{25} + 3x_{45} \geqslant x_{56} + x_{57} + x_{58} \\
(1-4)' \quad & x_{i,j} \in \{0, 1\} && (i, j) \in A
\end{aligned}
$$

Fig. 2 shows three feasible communication trees for this example with the associated total costs K:



(a) K = 36 (optimal) (b) K = 44 (c) K = 46

Fig. 2. Communication trees for the communication network of Fig. 1 with total costs K

The optimal solution of Fig. 2 (a) turns out to be a communication tree which uses the pure relay station nodes 2, 3 and 5. On the other hand, there is no pure relay station used in the tree of Fig. 2 (c). But the total cost K = 40 of the latter tree are higher compared with K = 36 of the optimal solution.

3.4 The solution of the optimization problem

The formulation of problem (1) implies that a cost-minimal communication tree can be computed by means of standard techniques of linear (0,1)-optimization. Unfortunately, this does not say too much about the efficiency of this way of solving the cost minimization problem. Though there has been an immense research going on in this field during the last ten years, it has not been possible to develop a powerful standard method for (0, 1)-optimization comparable in capability with the simplex method for linear programming. Thus, rather than applying standard algorithms it has turned out that in view of the combinatorial character of the linear (0, 1)-problem adapted algorithms based on the structure of the underlying optimization problem have to be found.

The authors have computational experience with three different types of algorithms. The first algorithm is a cutting-plane method. This approach takes advantage of the (+1, −1)-structure of the coefficients in a (0, 1)-formulation of the problem similar to (1–1) through (1–4). Especially, list-processing techniques are used in this algorithm to improve the efficiency of the simplex calculations in this special case [7]. The second algorithm is an implicit enumeration scheme [8]. The search in this algorithm is adapted to the cycle-free structure of the underlying graph. The typical advantage of this approach is that it allows to generate a great number of "near-optimal" alternative trees. The third algorithm, finally, is a kind of dynamic programming method [6] which derives solutions for subgraphs of G subsequently developed during the $n = |N|$ stages of the process. The back-tracking phase yields a cost-minimal total tree by combining these cost-minimal subtrees constructed in the preceding forward phase.

The efficiency of these three algorithms can be qualified by the criteria: maximal size of problems solvable, storage demands, and computing time.

The disadvantages of the cutting-plane method are the high demands of fast core memory and the unpredictable low convergence for ill-conditioned problems. The basic problem of the implicit enumeration algorithm is that the computation time grows nearly exponentially with the number of variables, i.e. the number of considered elementary ways of information transmission. This is due to the rather weak exclusion rules because normally there are many alternative near optimal solutions. The best computational experience was made with the third algorithm. In this dynamic programming algorithm the efficiency of the computations is strongly based upon the wordlength of the computer on which the algorithm is implemented. This is because all information about the computed subtrees is recorded and processed as bit-patterns.

Appendix

Example I: Organization of cost-minimal intro-company communication

The management of a company has changed the wage incentive payment plan in cooperation with a special committee of the works council. Now the details of this plan are to be transmitted to every pieceworker in the company. The management can use several different communication paths:

(1) It can transmit the plan directly to every pieceworker.

(2) It can use an indirect communication path by first informing either the two plant managers or the four foremen.

(3) The information can be transmitted by the special committee of the works council.

The feasible communication paths and the costs assigned to every path are shown on the following diagram.

Example II: Cost-minimal network of computer terminals as part of a management information system

A given number of computer terminals is to be installed. The displays are linked to the calculating unit by fixed wire connections.

A technical constraint allows only a connection between four displays and one display control unit.

A maximum of four display control units can be linked with a display junction unit. Up to four displays can have a direct connection with a display junction unit. Instead of a display another display control unit can be connected to a display control unit (i.e. there is a possibility of a cascading arrangement).

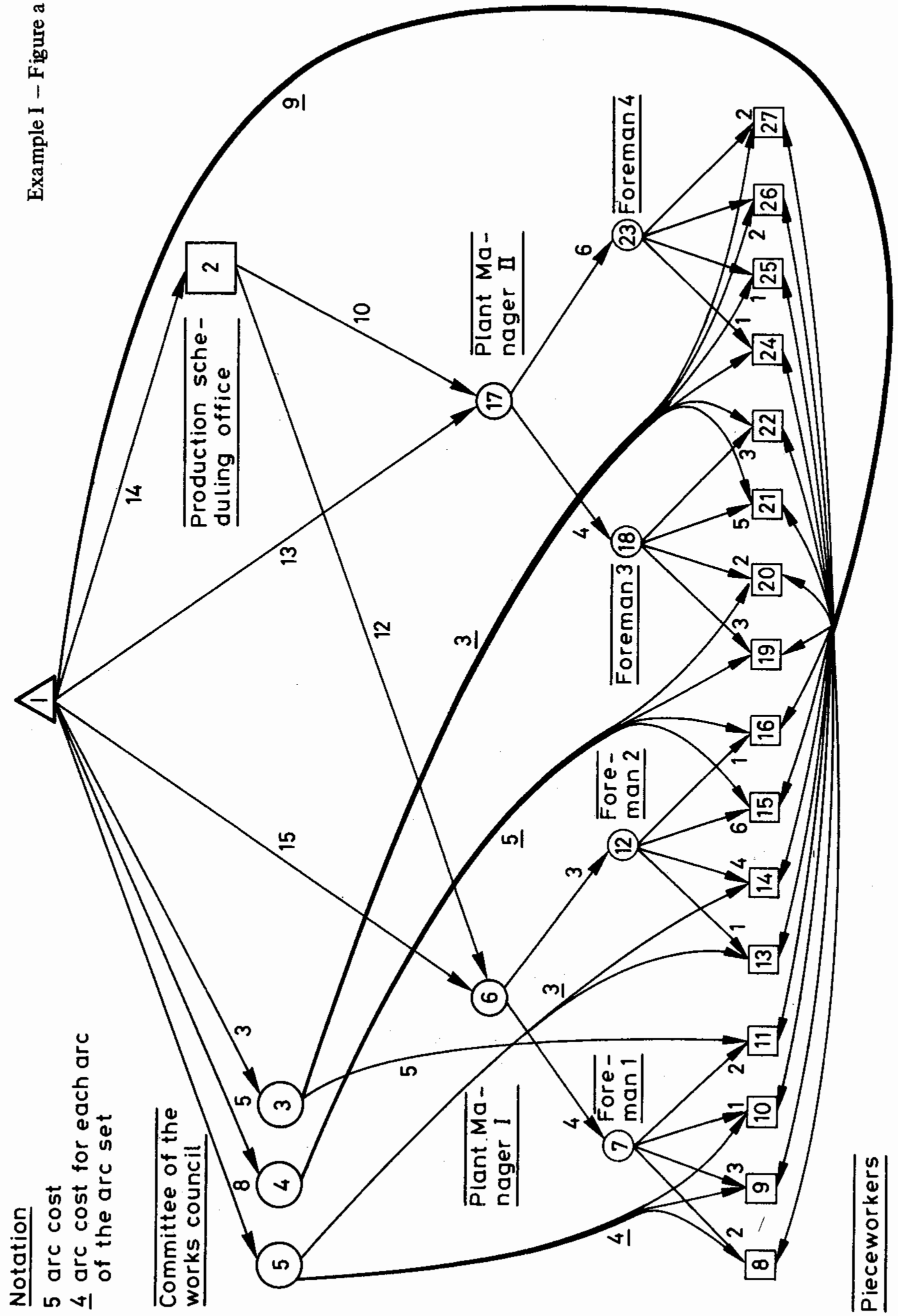
The feasible combinations of equipment produce cost consisting of

equipment and erection cost,

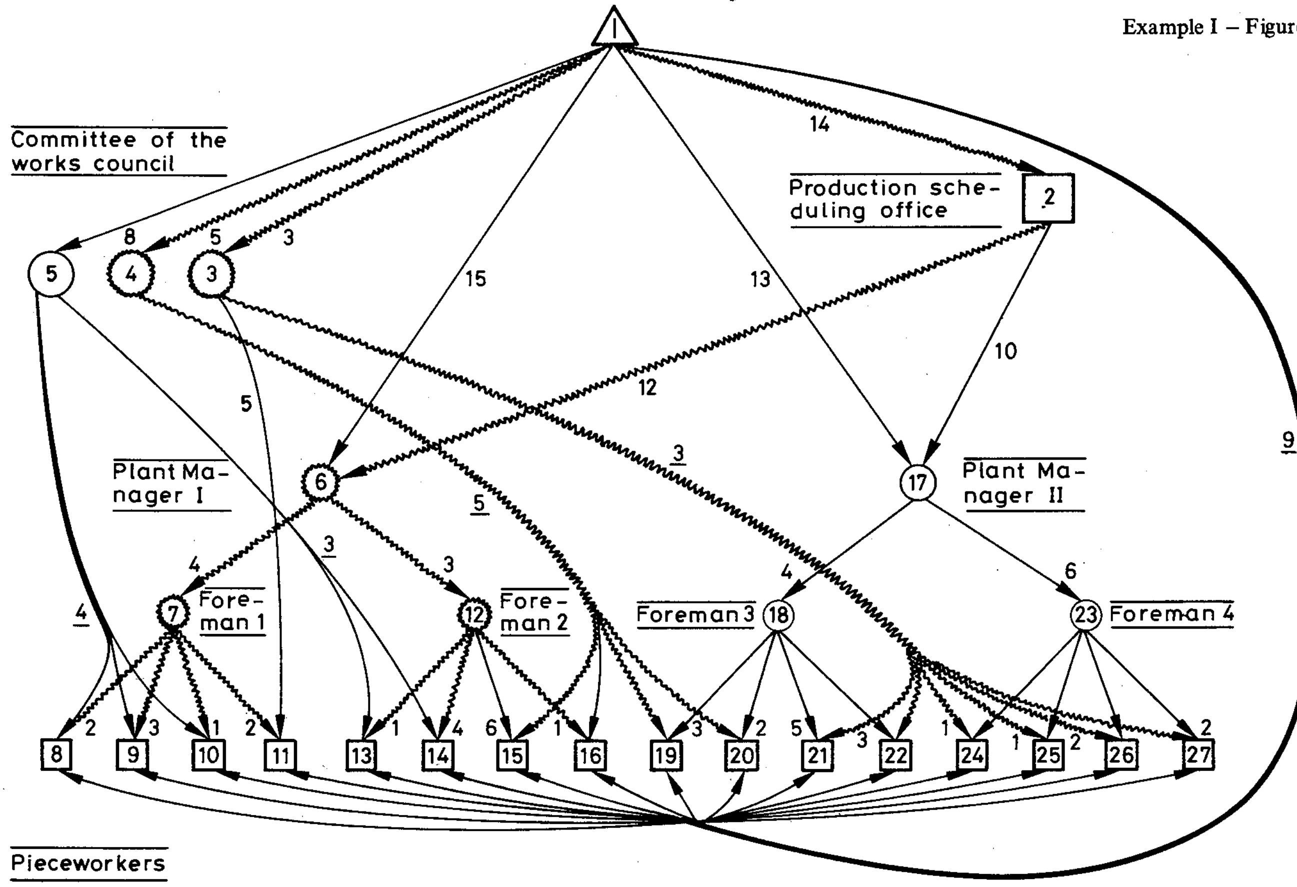
cost for material and initial expenditure for channels with different qualities and conditions of transmission.

The point is to find the cost-minimal cascading.

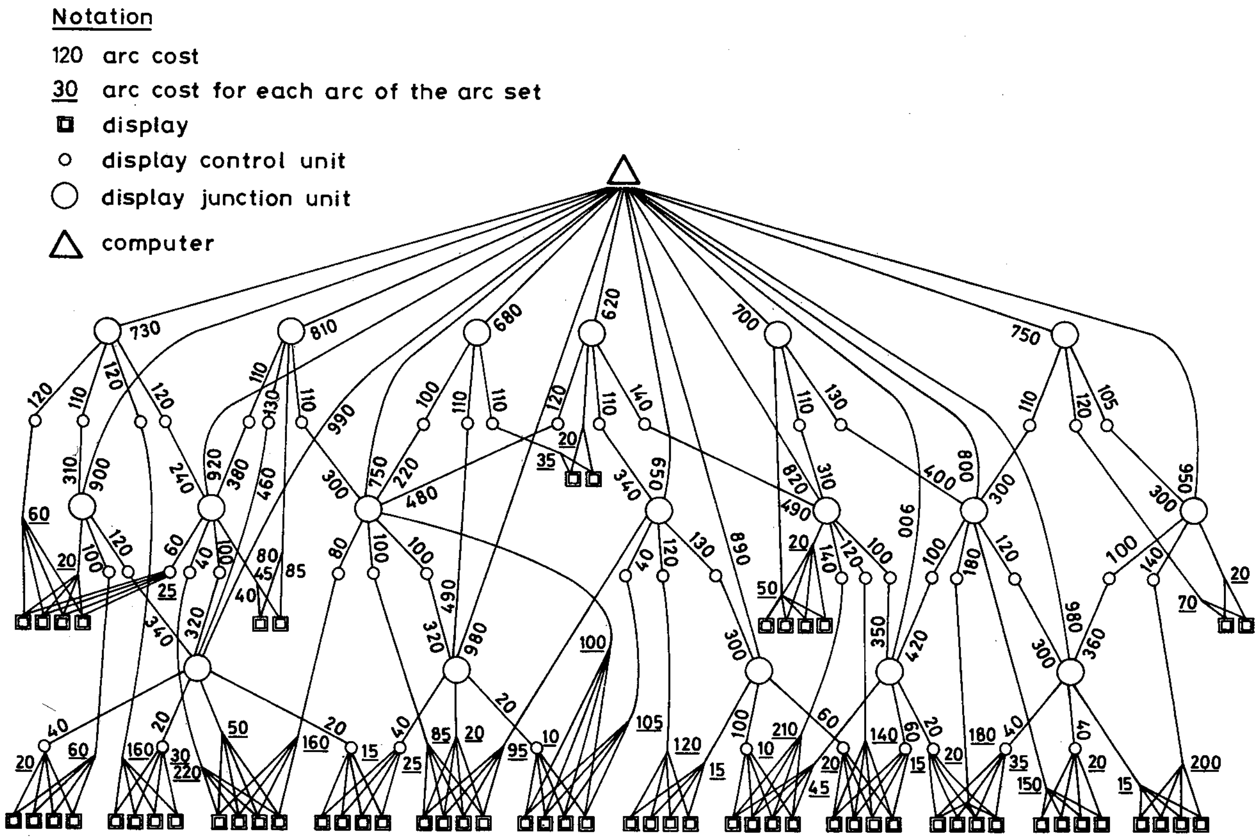
Example I – Figure a



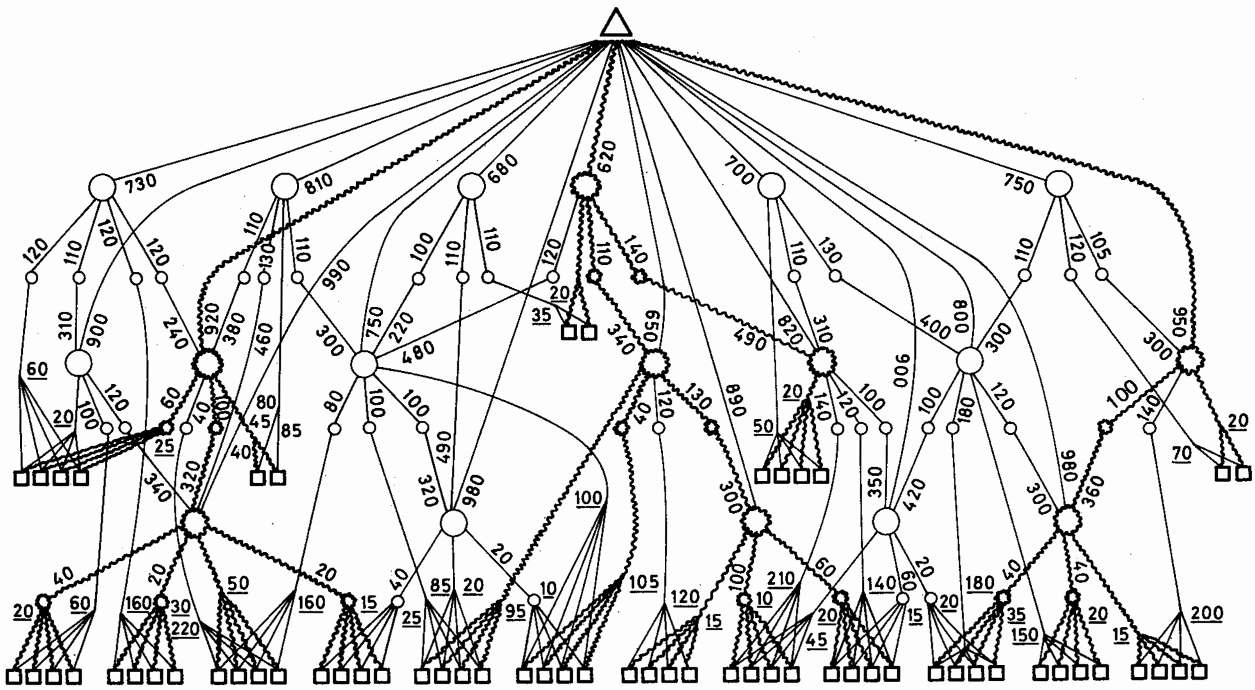
Example I – Figure b



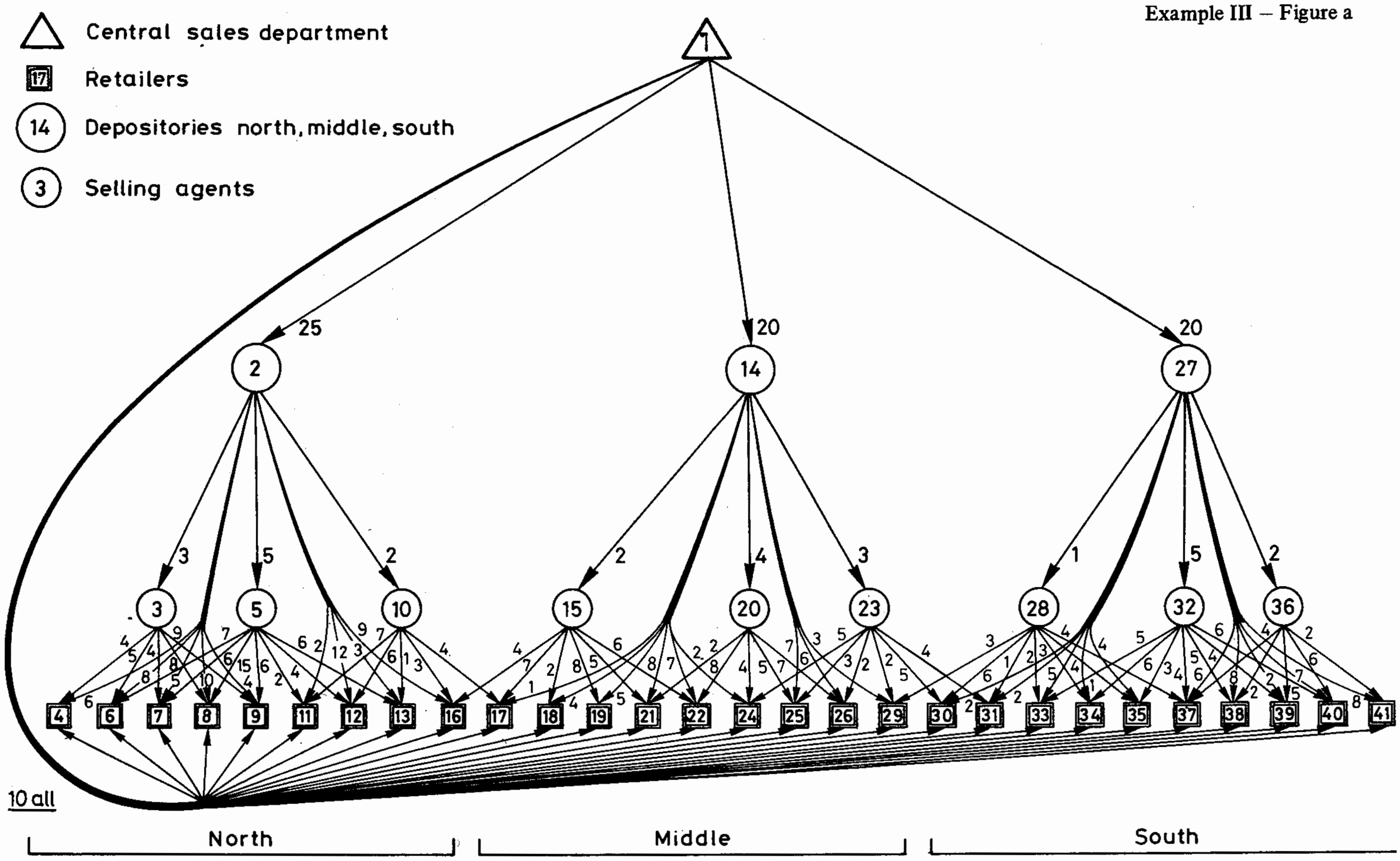
Example II – Figure a



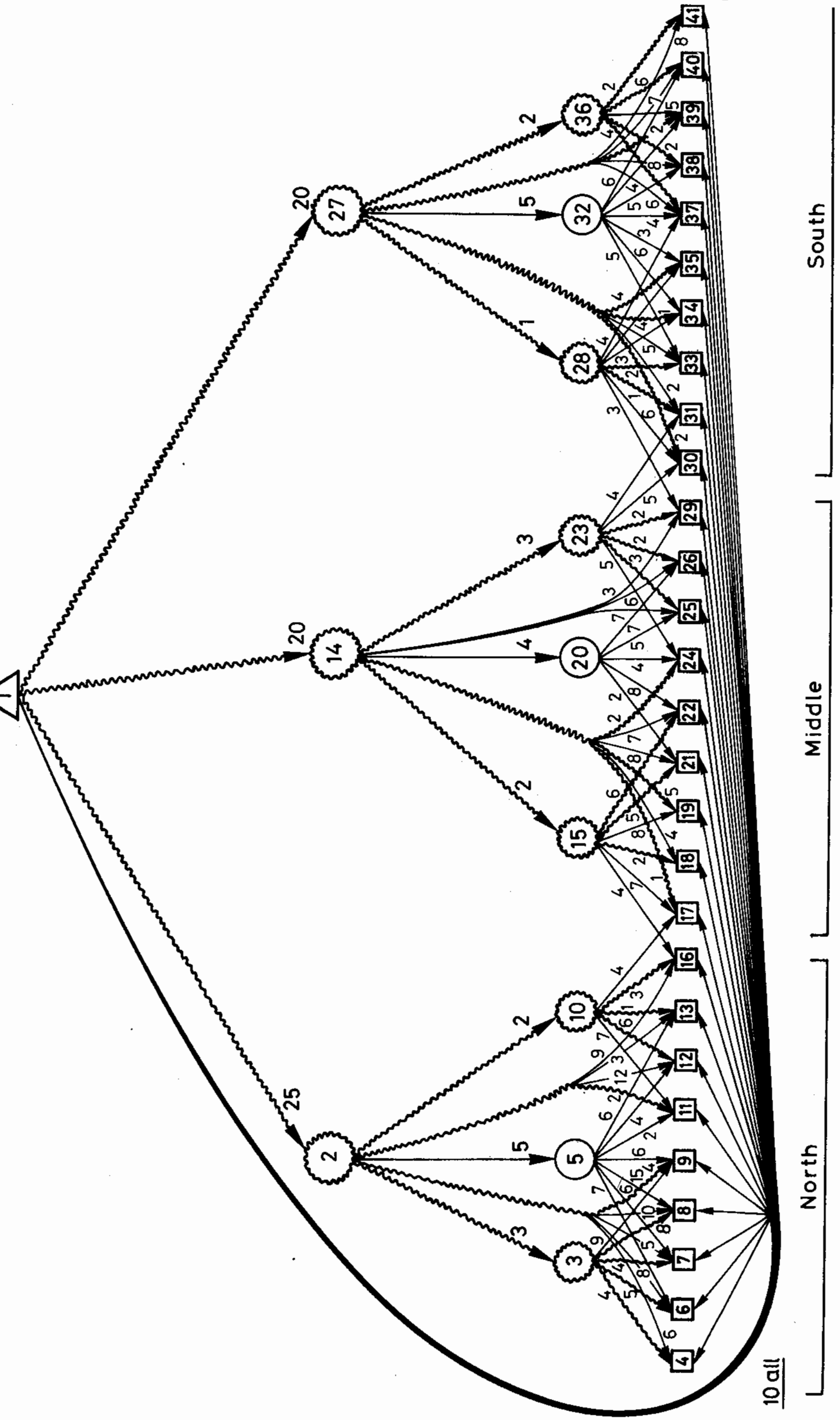
Example II – Figure b



Example III – Figure a



Example III – Figure b



Example III: Organization of cost-minimal introduction of a new product

The sales division of an enterprise wants to introduce a newly developed product. During the initial campaign a certain number of samples will be presented to some selected retailers in the three sales districts "North", "Middle" and "South". Also some information about the new product will be offered. The distribution problem can be solved in different ways:

(1) The central sales department mails the samples together with an instruction by letter. This alternative mainly produces delivery charges.

(2) The samples will be delivered to the decentral depositories "North", "Middle" and "South". From there the samples and a printed instruction will be distributed with the usual deliveries by car to the selected retailers. This alternative produces distribution cost and opportunity cost for storage capacity and loading space.

(3) The selling agents of the three decentral sales agencies "North", "Middle" and "South" offer the samples, a printed instruction and additional verbal information when they visit the retailers of their sales district. The selling agents can also visit retailers of an adjoining district, if these live near the districts border. This alternative produces travelling charges and cost of labour.

These alternatives are shown as different communication networks on the following diagram.

References

[1] Charnes, A., W.W. Cooper, R.J. Niehaus and A. Stedry: Static and Dynamic Assignment Models with Multiple Objectives, and Some Remarks on Organization Design. In: Management Science, Vol. 15, 1968/69, pp. B 365–375.

[2] Ford, L.R. and D.R. Fulkerson: Flows in Networks. Princeton N.J., 1962.

[3] Kern, W.: Die Netzplantechnik als Instrument betrieblicher Ablaufplanung. Anwendung der Netzplantechnik im Betrieb. Ed. by H. Jacob. Wiesbaden 1969.

[4] Kossbiel, H.: Kontrollspanne und Führungskräfteplanung, Grundlagen der betrieblichen Personalpolitik. Ed. by W. Braun, H. Kossbiel, G. Reber. Wiesbaden 1972.

[5] Müller-Hagedorn, L.: Grundlagen der Personalbestandsplanung. Opladen 1970.

[6] Nastansky, L.: Die Bestimmung kostenminimaler Bäume in gerichteten Netzwerken mit Hilfe der dynamischen Programmierung. Unpublished.

[7] Nastansky, L., S.M. Selkow and N.F. Stewart: The Enumeration of Minimal Phylograms. In: Bulletin of Mathematical Biophysics, Vol. 35, 1973, pp. 525–533.

[8] Nastansky, L., S.M. Selkow and N.F. Stewart: Cost-Minimal Trees in Directed Acyclic Graphs. In: Zeitschrift für Operations Research, Vol. 18, 1974, pp. 59–67.

[9] Wild, J.: Neuere Organisationsforschung in betriebswirtschaftlicher Sicht. Berlin 1967.

German abstract
Zur Wahl des gesamtkostenminimalen Kommunikationssystems

Von *Ludwig Nastansky* und *Hans Jürgen Drumm*

Der Beitrag beschäftigt sich mit dem Problem, ein kostenminimales Kommunikationssystem in einem Kommunikationsnetzwerk zu bestimmen. Die Ermittlung kostenminimaler Kommunikationssysteme gehört zu den sogenannten Aufbauproblemen der Organisationsplanung. Solche Probleme können häufig mit Flußmodellen und kombinatorischen Algorithmen in Netzwerken gelöst werden. Hier wird ein Modell vorgeschlagen, das mit Hilfe eines kombinatorischen Algorithmus lösbar ist.

Das Kommunikationsnetzwerk enthält eine Gruppe von Informationsquellen, von denen Informationen an eine Gruppe von Informationsempfängern zu übermitteln sind. Neben Quellen und Empfängern gibt es noch reine Relaisstellen, die Informationen empfangen und weiterleiten können; diese Relaisstellen können für die Informationsweitergabe benutzt werden, müssen es aber nicht. Quellen, Empfänger und Relaisstellen definieren Knoten in einem gerichteten Kommunikations-Netzwerk.

Die Kanten des Netzwerks kennzeichnen elementare Informationswege; elementar bedeutet: von einer Sendestelle (Ausgangsknoten) zu einer empfangenden Stelle (Endknoten). Mögliche elementare Informationswege sind: Quelle nach Empfänger, Quelle nach Relaisstelle, Relaisstelle nach Relaisstelle, Relaisstelle nach Empfänger. Jedem dieser elementaren Informationswege sind Kosten für die Übertragung der Information zugeordnet. Die Besonderheit dieses Problems besteht darin, daß auch bei wiederholtem Abruf von Informationen aus einem Knoten keine zusätzlichen Kosten für die Übertragung von Informationen zu diesem Knoten auftreten. Dadurch unterscheidet sich dieses Problem von Standard-Flußmodellen, bei denen jede von einem Knoten ausgehende Flußeinheit auch zuvor zu diesem Knoten hintransportiert werden muß.

In einer konkreten Anwendungssituation wird häufig eine Vielzahl von Alternativen bestehen, um die notwendigen Informationen von den Quellen zu allen Empfängern zu übertragen. Ein wichtiges Problem ist in diesem Zusammenhang, denjenigen Kommunikationsbaum in dem vorliegenden Kommunikationsnetzwerk zu bestimmen, der bei geringsten Gesamtkosten des Kommunikationssystems garantiert, daß jeder Empfänger von den Quellen mit Informationen beliefert wird. Die Frage ist hier insbesondere, ob und welche Relaisstellen man einschalten soll.

Für die Ermittlung des kostenminimalen Kommunikationsbaumes innerhalb eines gegebenen Kommunikationssystems wurde ein dynamischer Programmierungs-

algorithmus entwickelt, der als Fortran-Programm verfügbar ist. Anwendungsfälle des Algorithmus werden an drei Beispielen aufgezeigt: Organisation der kostenminimalen innerbetrieblichen Benachrichtigung, kostenminimales Terminalnetz im Rahmen eines Informationssystems mit Kaskadierung und Organisation der kostenminimalen Produkteinführung.